International Journal of Heat and Fluid Flow 23 (2002) 74–80

International Journal of HEAT AND FLUID FLOW

www.elsevier.com/locate/ijhff

Spin-up of a liquid with a density maximum in a cylinder

Chang Ho Lee [a], Jae Min Hyun [a,*], Ho Sang Kwak [b]

[a] Department of Mechanical Engineering, Korea Advanced Institute of Science & Technology, 373-1 Kusong-dong, Yusong-gu, Taejon 305-701, Republic of Korea

[b] School of Mechanical Engineering, Kumoh National University of Technology, 188 Sinpyeong-dong, Kumi-Shi, Kyeongsangbuk-do 730-701, Republic of Korea

Received 15 December 2000; accepted 24 July 2001

Abstract

A study is made of the spin-up of a viscous non-Boussinesq fluid in a vertically mounted cylinder. The density (ρ) of the fluid becomes maximum at temperature T_{m}, and a quadratic density–temperature relation is used. The fluid is stratified by imposing a vertical temperature contrast, with the temperature at the bottom endwall disk being T_{m}. Comprehensive numerical solutions to the time-dependent Navier–Stokes equations are acquired. Due to the vertically non-uniform stratification, spin-up proceeds faster near the bottom endwall than near the top endwall. The meridional circulation is more intense near the bottom endwall. Detailed descriptions of evolutions of both azimuthal and meridional flows are given. Major differences in dynamic characteristics are illustrated between a homogeneous fluid, a Boussinesq fluid, and the present non-Boussinesq fluid which has a density maximum. © 2002 Elsevier Science Inc. All rights reserved.

1. Introduction

Spin-up refers to the transient adjustment of an enclosed fluid from a state of solid-body rotation, subject to a change in rotation rate of the container. Specifically, a closed cylinder [radius R and height H, aspect ratio $H/R \sim \mathrm{O}(1)$], filled with a viscous incompressible fluid and rotating steadily about its axis at angular velocity Ω_i, is considered. At the initial instant $t = 0$ the angular velocity of the container is abruptly increased to Ω_f [$\equiv \Omega_i + \Delta\Omega, \varepsilon \equiv \Delta\Omega/\Omega_i$], and the task is to describe the transient response of the fluid.

The classical treatise of Greenspan and Howard (1963) carried out a linearized analysis for the case $\varepsilon \ll 1$ of a homogeneous fluid. The important non-dimensional parameter is the Ekman number E [$\equiv 4\nu/\Omega_i H^2$], in which ν denotes the kinematic viscosity of fluid. In most technological applications, $E \ll 1$, which implies that direct effects of viscosity are confined to the boundary layers near the solid walls. It was demonstrated that the main dynamical element is the meridional circulation, which is driven by the suction of Ekman boundary layers at the endwall disks. In the inviscid interior, angular momentum is conserved, and the radially inward meridional circulation effectuates the increase of angular velocity at a given location. Therefore, the overall fluid adjustment to the altered rotation rate of the container is substantially accomplished over the spin-up timescale $\mathrm{O}(E^{-1/2}\Omega_i^{-1})$, which is order-of-magnitude smaller than the diffusive timescale $\mathrm{O}(E^{-1}\Omega_i^{-1})$. For a homogeneous fluid, the evolution of angular velocities in the interior is uniform in the axial direction. The essentials of the model of Greenspan and Howard have now been firmly established (Warn-Varnas et al., 1978; Dolzhanskii et al., 1992).

Spin-up of a stably stratified fluid in a vertically mounted cylinder, the sidewall of which is insulated, brings forth an additional dynamical element (Walin, 1969; Sakurai, 1969). The Ekman-layer suction is still present, but, due to the inhibition of vertical velocities by stratification, the penetration of meridional circulation is limited to the interior regions close to the endwall disks. Spin-up proceeds in a spatially non-uniform manner in the interior, and the overall rate of change of angular velocity in the interior is slower than for the homogeneous fluid. These qualitative findings for the

* Corresponding author. Tel.: +82-42-869-3012; fax: +82-42-869-3210.

E-mail address: jmhyun@cais.kaist.ac.kr (J.M. Hyun).

0142-727X/02/$ - see front matter © 2002 Elsevier Science Inc. All rights reserved.
PII: S0142-727X(01)00133-3

case of a stratified fluid carry significant implications in geophysical and industrial applications.

In the numerical and experimental programs on stratified spin-up (Sakurai, 1969; Hyun et al., 1982), the customary Boussinesq-fluid assumption has been invoked, i.e.,

$$\rho = \rho_B[1 - \alpha(T - T_B)], \quad (1)$$

in which the linear density (ρ) – temperature (T) relation is postulated. In the above, subscript B refers to the reference value, and α the coefficient of thermometric expansion. The linear ρ–T relation of Eq. (1), however, is not applicable to certain liquids in the neighborhood of a specific temperature T_m at which density reaches a maximum ρ_m. The best known example is water, which has maximum density ρ_m at $T_m = 3.98$ °C . This non-linear ρ–T behavior calls for a new dynamical consideration in the discussion of buoyancy-related convections. A substantial body of literature exists on the behavior of a non-Boussinesq fluid in a non-rotating environment (Robillard and Vasseur, 1982; Braga and Viskanta, 1992; Nishimura et al., 1995; Kwak et al., 1998), but published works are scarce on rotating flows of fluid near its density maximum.

The present paper intends to address the spin-up process in a cylinder of a fluid with a density maximum. In the present endeavor, such a fluid will be referred to as a density-maximum fluid. For definiteness, the temperature at the bottom endwall disk is T_B [$= T_m$], the temperature of maximum density, and the temperature at the top endwall disk T_T is higher than T_m, i.e., $\Delta T \equiv T_T - T_B > 0$. Numerical solutions to the governing Navier–Stokes equations are secured, and emphasis will be placed on delineating the differences in the transient flow characteristics between a usual Boussinesq-fluid and a density-maximum fluid of present concern.

2. Formulation

As remarked earlier, the (ρ)–(T) relationship near T_m of a density-maximum fluid is modeled by a parabolic function (Moore and Weiss, 1973):

$$\rho = \rho_m\left\lfloor 1 - \beta(T - T_m)^2 \right\rfloor. \quad (2)$$

In the case of water, the error associated with Eq. (2), with $\beta = 8.0 \times 10^{-6}$ (°C)$^{-2}$, $T_m = 3.98$ °C, is smaller than 4% in the range from 0 to 8 °C. In the present problem setting, the entire temperature range is assumed to lie within the bound in which Eq. (2) is valid. The other thermophysical properties of the fluid are taken to be constant at T_m (Robillard and Vasseur, 1982; Kwak et al., 1998).

At the initial state, both the cylinder and fluid rotate steadily at the rotation rate Ω_i. With ΔT [$\equiv T_T - T_B$] > 0, with $T_B = T_m$, a stable stratification prevails. The geometrical layout, together with the boundary conditions, is sketched in Fig. 1.

The governing non-dimensional time-dependent axisymmetric Navier–Stokes equations, written in the cylindrical frame (r, ϕ, z) rotating at Ω_i with corresponding velocity components (u, v, w), are

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \quad (3a)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \left(2 + \frac{v}{r}\right)v = -\frac{\partial p}{\partial r} + \frac{E}{4}\left[\nabla^2 u - \frac{u}{r^2}\right], \quad (3b)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \left(2 + \frac{v}{r}\right)u = \frac{E}{4}\left[\nabla^2 v - \frac{v}{r^2}\right], \quad (3c)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{E}{4}\nabla^2 w + S_N\theta^2, \quad (3d)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial r} + w\frac{\partial \theta}{\partial z} = \frac{E}{4Pr}\nabla^2\theta, \quad (3e)$$

where

$$\nabla^2 = \frac{\partial}{r\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}.$$

In the above, non-dimensionalization was performed by adopting H, $H\Omega_i$, and $\rho_m(H\Omega_i)^2$ as scales for length, velocity and pressure, respectively. The non-dimensional temperature θ is defined as $\theta \equiv (T - T_m)/(T_T - T_m)$. The Prandtl number Pr $\lfloor \equiv \nu/\kappa \rfloor$, where κ is the thermal diffusivity of fluid, is assumed as O(1); the stratification

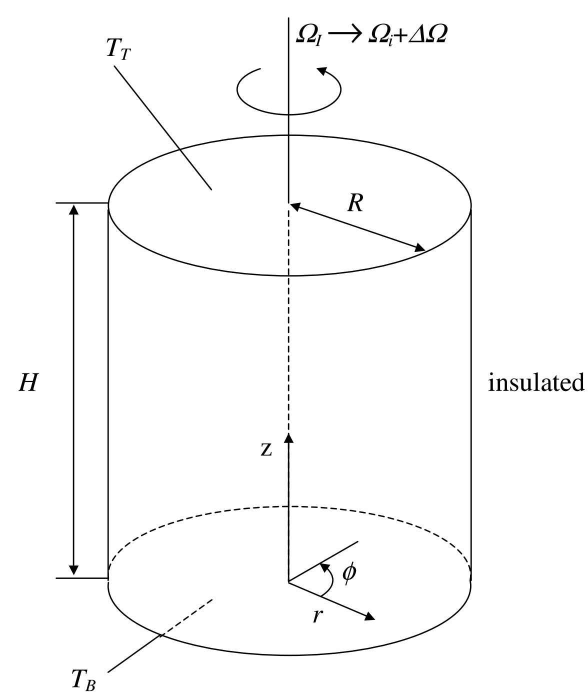

Fig. 1. Flow layout and coordinate system.

number S_N [$\equiv \beta g(T_T - T_m)^2/H\Omega_i^2$] represents the overall buoyancy effect relative to the effect of rotation. For comparison purposes, for a Boussinesq fluid of Eq. (1), the last term of Eq. (3d), $S_N\theta^2$, should be replaced by $S_B\theta$, in which $S_B \equiv [\alpha g(T_T - T_m)/H\Omega_i^2]$. Both S_N and S_B may be recast as $[g(\Delta\rho/\rho_B)/H\Omega_i^2]$, in which $\Delta\rho/\rho_B$ represents the density difference between the bottom and top endwall disks, relative to the reference value at the bottom disk.

As stated earlier, at the initial state, the fluid is motionless with a vertically linear temperature profile:

$$u = v = w = 0, \ \frac{\partial\theta}{\partial z} = 1 \quad \text{at } t = 0. \tag{4}$$

It is noted that, because of the quadratic ρ–T relation, the vertical distribution of density is not linear.

The boundary conditions, which reflect the abrupt change in rotation rate of the cylinder from Ω_i to $\Omega_i(1+\varepsilon)$, are expressed as

$$\begin{aligned} &u = w = 0, \ v = r\varepsilon, \ \theta = 0 \quad \text{at } z = 0,\\ &u = w = 0, \ v = r\varepsilon, \ \theta = 1 \quad \text{at } z = 1,\\ &u = w = 0, \ v = \frac{R}{H}\varepsilon, \ \frac{\partial\theta}{\partial r} = 0 \quad \text{at } r = R/H,\\ &u = v = \frac{\partial w}{\partial r} = 0, \ \frac{\partial\theta}{\partial r} = 0 \quad \text{at } r \to 0. \end{aligned} \tag{5}$$

A point to be addressed here is the assumption of axisymmetry of flow. The preceding studies on linear stratified spin-up indicated that, when the Rossby number is small $\varepsilon \lesssim 0.2$, the flow is axisymmetric to a high degree of accuracy. The laboratory experiments of Buzyna and Veronis (1971), Saunders and Beardsley (1975) and numerical computations of Barcilon et al. (1975) consistently demonstrated the flow axisymmetry for $10^{-4} \lesssim E \lesssim 10^{-3}$, $\varepsilon \lesssim 0.2$, and stratification number O(1). The early theoretical accounts of Walin (1969) and Sakurai (1969) on linear stratified spin-up also invoked the assumption of flow axisymmetry. It is mentioned that, in the case of nonlinear spin-up from rest of a stratified fluid in a cylinder, the exploratory experiment of Greenspan (1980) showed that the flow is axisymmetric until moderate times and non-axisymmetry sets in in the later stages.

As stated previously, this paper aims to depict the linear spin-up flows of a stratified fluid from an established rigid-body rotation. In view of the aforesaid experimental observations, it is reasonable to treat axisymmetric flows in the present parameter range. In addition, it is emphasized that the present axisymmetric flows serve as the basic-state. A formal stability analysis can be performed to determine the onset of non-axisymmetry by perturbing this axisymmetric basic-state flows. In a related problem formulation, descriptions of the basic-state axisymmetric flow and of its stability property due to centrifugal forces were attempted recently (e.g., Park and Hyun, 2001). In their endeavors, a portrayal of the axisymmetric flow constitutes the first step toward the analysis of more realistic situations.

The numerical solution procedure is based on the widely used SIMPLER algorithm (Patankar, 1980), together with the QUICK scheme (Hayase et al., 1992). The mesh was stretched to cluster grid points near the boundaries of the computational domain. At least five grid points were placed inside the Ekman layer. Most of the calculations were conducted by deploying a network of (61×81) staggered grid points in the (r–z) plane. The computational time increment was typically $\Delta t = 0.01$. Convergence was declared when the maximum relative difference between successive iteration levels fell below 10^{-6}. Extensive grid- and time step-convergence tests were carried out by repeating calculations of a large number of previously documented exemplary cases (Warn-Varnas et al., 1978; Hyun et al., 1982). The outcome was highly mutually consistent. The difference between the results based on the (61×81) and (91×121) networks was shown to be less than 0.1%. Time stepping was based on an implicit scheme with first-order accuracy in (Δt).

3. Results and discussion

Numerical results are now analyzed. Attention was focused on the qualitative changes in flow characteristics as the stratification number S_N is altered. For all the results reported here, Pr was set to be $Pr = 11.573$, which simulates water at $T = 3.98$ °C, and $H/R = 1.0$. The Rossby number ε and Ekman number E were fixed at very small values, 0.01 and 4×10^{-4}, respectively, to compare with the linear spin-up results for a Boussinesq fluid (Walin, 1969; Sakurai, 1969; Hyun et al., 1982).

An exemplary set is displayed in Fig. 2, which shows the progress of spin-up at different depths on the mid-radius ($r = \frac{1}{2}R/H$). The azimuthal velocity v is normalized by $r\varepsilon$. Time is scaled by using the homogeneous spin-up timescale $E^{-1/2}$, which was ascertained in the classical analysis of Greenspan and Howard (1963). Fig. 2(a) exhibits the case of a homogenous-fluid spin-up. Clearly, the rate of increase of the scaled angular velocity $v/r\varepsilon$ is fairly uniform in the vertical direction. In the interior region, i.e., the region far from the solid walls of cylindrical container, the dominant mechanism of spin-up process for a homogeneous fluid is inviscid in nature, and angular momentum is conserved. As elucidated by Greenspan and Howard, the suctions of Ekman boundary layers at both endwall disks induce radially inward meridional circulation in the interior. The angular velocity of a fluid parcel moving radially inward increases with time as the radial arm shortens. In the case of a homogeneous fluid, this radially inward flow is vertically uniform in the interior and, therefore, the spin-up process is vertically uniform. Fig. 2(b) exemplifies the

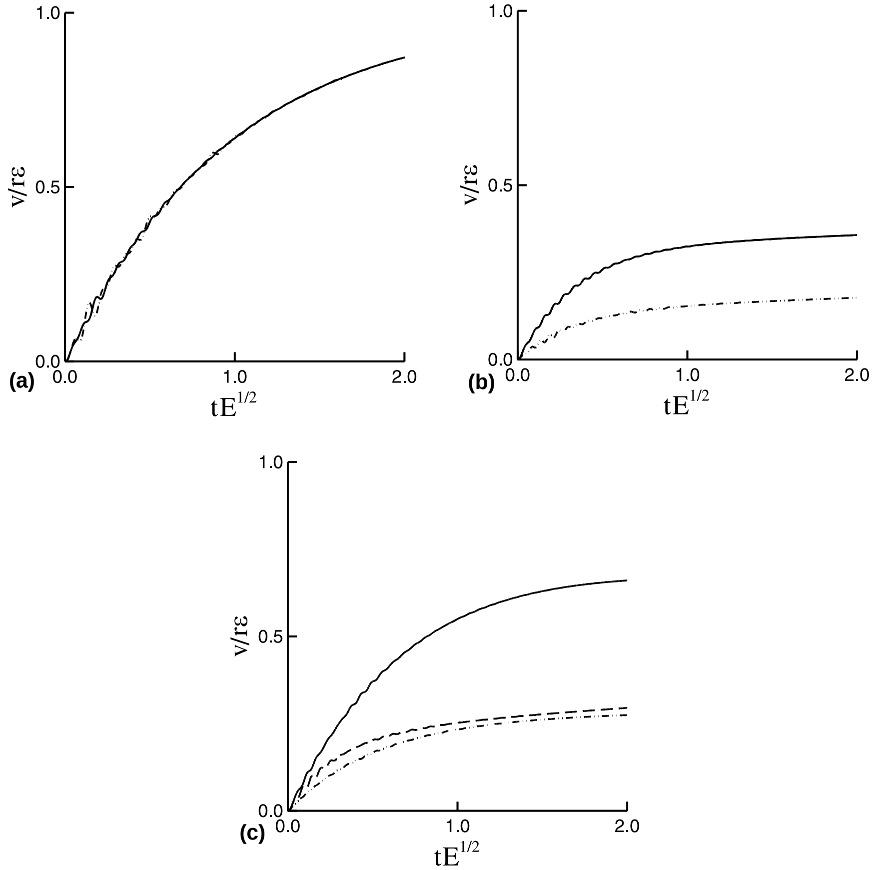

Fig. 2. Evolution of normalized azimuthal velocity, $v/r\varepsilon$ at mid-radius, $r = 0.5R/H$. (a) Homogeneous fluid; (b) Boussinesq fluid ($S_B = 10.0$); (c) density-maximum fluid ($S_N = 10.0$). Vertical locations are: (——) $z = 0.25$; (– · ·–) $z = 0.5$; (– – –) $z = 0.75$.

spin-up process of a stratified, Boussinesq fluid ($S_B = 10$). The rate of increase of $v/r\varepsilon$ is slower than for a homogeneous fluid. Since the prevailing stratification is uniform throughout the cylinder, $v/r\varepsilon$ is symmetric about the mid-height ($z = 0.5$). [In Fig. 2(b), the curves for $z = 0.25$ and $z = 0.75$ overlap.] Here, $v/r\varepsilon$ at mid-height ($z = 0.5$) is smaller than at $z = 0.25$. This feature is explained by noting that, in the case of a stratified, Boussinesq fluid, the vertical motion is restricted by the stabilizing density stratification, and, therefore, the meridional circulation is restricted to the vicinities of the endwalls. The case of a stratified, density-maximum fluid ($S_N = 10$) is demonstrated in Fig. 2(c). The value of $v/r\varepsilon$ at $z = 0.25$ is larger than those at $z = 0.5$ and at $z = 0.75$. Recall that, due to the quadratic (ρ–T) relationship of Eq. (2), density stratification in the cylinder increases with height. Near the bottom endwall, the meridional circulation is intense, and the spin-up proceeds at a faster rate. Near the top endwall, the meridional circulation is weaker than near the bottom endwall, because density stratification is stronger, and, consequently, spin-up proceeds at a slower rate. At the mid-height, penetration of meridional circulation from both endwalls is meager due to the presence of stable stratification. The rate of spin-up, therefore, is slowest in the mid-height region.

In conjunction with the depiction of evolutions of azimuthal velocity v, it is useful to monitor the time-dependent meridional flow patterns. Here, the meridional stream function ψ is defined such that $u = (1/r)(\partial\psi/\partial z)$ and $w = -(1/r)(\partial\psi/\partial r)$. As succinctly captured in the conceptual model of Greenspan and Howard (1963), the meridional flows undergo distinctive stages. Within the rotational timescale of $O(\Omega_i^{-1})$, the Ekman layers form and, throughout the major adjustment phase of timescale $O(E^{-1/2}\Omega_i^{-1})$, the Ekman layers are thought to have been established instantaneously. The Ekman pumping is vigorous at small times, which pushes the meridional circulations toward the mid-height regions. As the interior fluid is spun up, the difference in rotation rate between the interior fluid and the endwall is reduced. This points to attenuated Ekman layer pumping at intermediate and large times. As the fluid system approaches the new steady state, the Ekman pumping weakens further and the meridional flows diminish. These qualitative evolutionary features are discernible in Fig. 3, and the effects of stratification, both for Boussinesq and density-maximum fluids, are clearly illustrated. For a homogeneous fluid [see column (a) of Fig. 3], the meridional circulations, which are anti-symmetric with respect to the mid-height ($z = 0.5$), fill

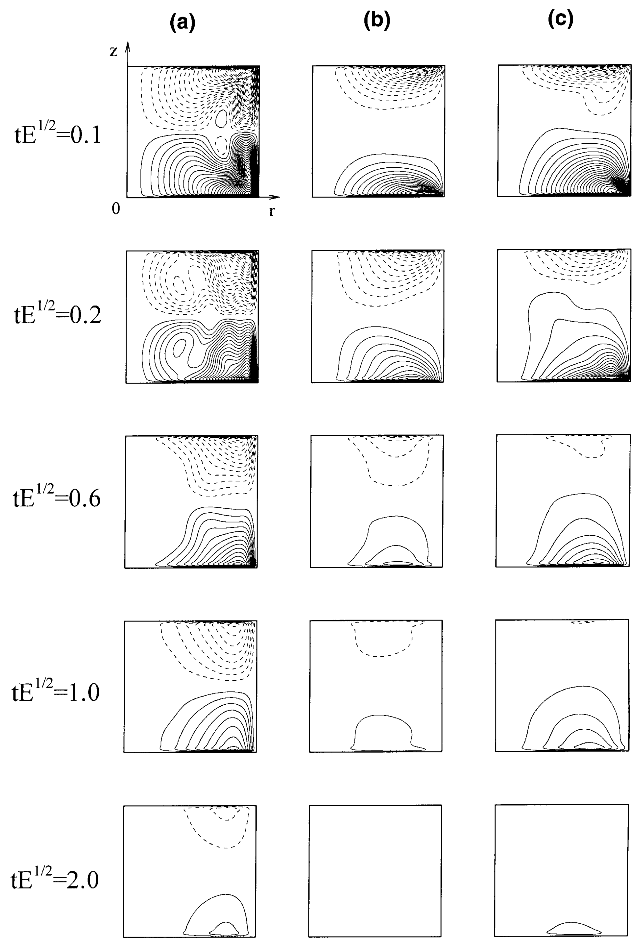

Fig. 3. Sequential plots of meridional stream function ψ. (a) Homogeneous fluid; (b) Boussinesq fluid, $S_B = 7.0$; (c) density-maximum fluid, $S_N = 7.0$.

much of the entire cylinder. In the top (bottom) region of the cylinder, a clockwise (counter-clockwise) meridional circulation cell forms. This arises by the Ekman-layer suction of fluid toward the disk in the central axial region. In the bulk of interior core, the radially inward meridional motions are substantially uniform in the axial direction. Since the angular momentum is conserved in the interior, the azimuthal velocity increases as the radial arm of a fluid particle is reduced. In the case of a Boussinesq fluid, stratification is uniform; therefore, the meridional motions are still anti-symmetric about the mid-height ($z = 0.5$). However, the prevailing stratification inhibits vertical motions, which causes the meridional circulations to be concentrated in the regions close to the endwalls. As is evident in the center-column plots of Fig. 3, meridional motions are less intense in the mid-height region. These are reflected in an axially non-uniform rate of spin-up, and spin-up is retarded most in mid-height regions. These observations are consistent with the evolutions of v-velocity displayed in Fig. 2. The right column of Fig. 3 exhibits the sequential pictures of meridional flows for a density-maximum fluid. Since the prevailing stratification increases with height, the degree of inhibition of vertical motions becomes more effective in the top regions of the cylinder. Consequently, at intermediate and large times, the degree of attenuation of the clockwise (counter-clockwise) circulating meridional cell near the top (bottom) endwall disk is comparatively more (less) effective. The breakup of this symmetry is attributable to the axially non-uniform distribution of prevailing stratification, which stems from the nonlinear ρ–T relationship of a density-maximum fluid. As displayed in Fig. 2, the spin-up of fluid in the bottom region (e.g., at $z = 0.25$) proceeds faster than in the top region (e.g., at $z = 0.75$).

The behavior of meridional flow, as influenced by stratification, can further be appreciated in the z-variation plots of u and w of Fig. 4. For a homogeneous fluid, large radially outward flows exist in the Ekman boundary layers, and in the interior, radially inward motions are seen, and these are fairly uniform in the axial direction. The axial flows are anti-symmetric about

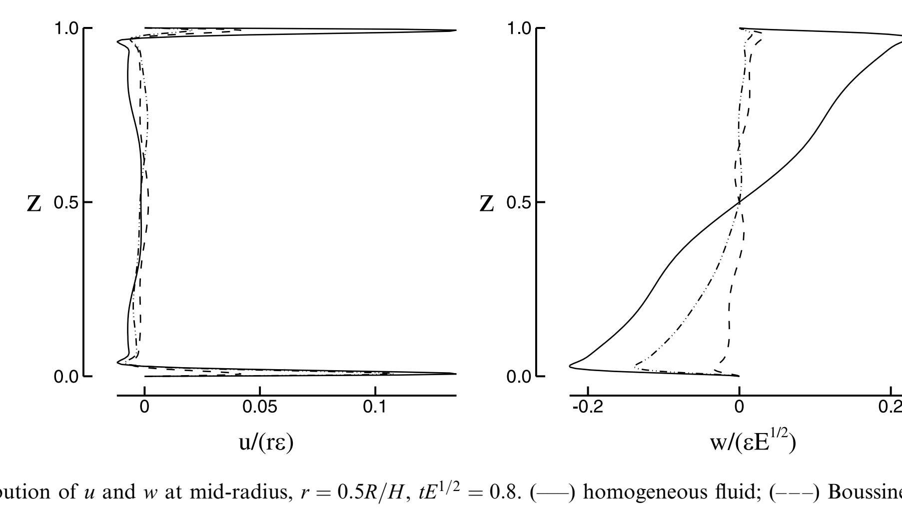

Fig. 4. Vertical distribution of u and w at mid-radius, $r = 0.5R/H$, $tE^{1/2} = 0.8$. (——) homogeneous fluid; (– – –) Boussinesq fluid ($S_B = 10$); (– · –) density-maximum fluid ($S_N = 10$).

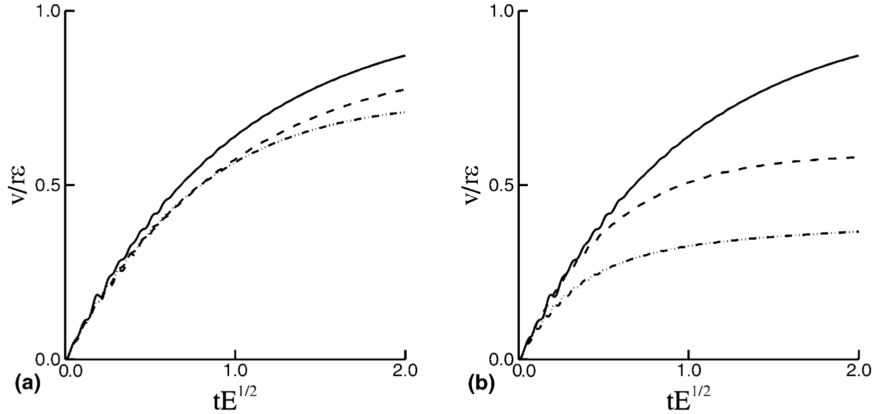

Fig. 5. Effects of stratification on the evolution of $v/r\varepsilon$, $r = 0.5R/H$: (a) $z = 0.25$; (b) $z = 0.75$. (——) $S_N = 0$; (– – –) $S_N = 3.0$; (– · ·–) $S_N = 7.0$.

mid-height. For a Boussinesq fluid, the anti-symmetry of w about mid-height is largely preserved. Inhibition of w in much of the cylinder is visible. In the case of a density-maximum fluid, the suppression of w is pronounced in the top region of the cylinder. In the bottom region, the inhibition of w is less ostensible due to the relatively weak stratification in this region. These observations reinforce the prior physical interpretations of Figs. 2 and 3.

For a density-maximum fluid, Fig. 5 illustrates the impact of the strength of overall stratification on the rate of spin-up. As remarked previously, spin-up proceeds in axial uniformity for a homogeneous fluid [see curves for $S_N = 0$]. The retardation of spin-up by the introduction of stratification is more pronounced in the top region of the cavity. This is in line with the preceding assertion that, for a given value of S_N, stratification is stronger in the top region, which gives rise to a further suppression of meridional motions in this region. The results in Fig. 5 demonstrate that spin-up, in terms of v, is affected less in the bottom region of the cylinder when stratification is imposed.

In view of the spatial non-uniformity of spin-up, it is useful to measure the global rate of spin-up for the entire fluid in the cavity. The volume-averaged angular velocity $\overline{\Omega}$ is introduced for this purpose:

$$\overline{\Omega} = 2\left(\frac{H}{R}\right)^2 \int_0^1 \int_0^{R/H} \left(\frac{v}{r}\right) r\,\mathrm{d}r\,\mathrm{d}z. \tag{6}$$

Comparisons are made in Fig. 6 to gauge the evolutions of $\overline{\Omega}$. Obviously, the overall spin-up proceeds fastest for a homogeneous fluid. It is noted that, for the same overall strength of stratification, $\overline{\Omega}$ is larger for a density-maximum fluid than for a Boussinesq fluid. This is due to the non-uniform distribution of prevailing stratification for a density-maximum fluid, in which the suppression of meridional motions is comparatively less severe in the bottom region of the cavity.

By replotting the numerical results of $\overline{\Omega}$, the spin-up time τ_s, normalized by the homogeneous-fluid spin-up

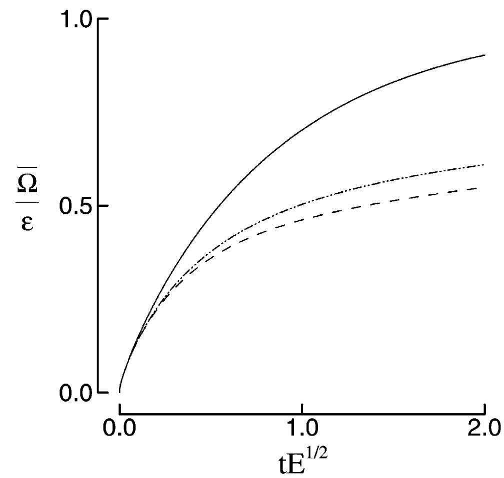

Fig. 6. Time evolution of the volume-averaged angular velocity $\overline{\Omega}$. (——) Homogeneous fluid; (– – –) Boussinesq fluid ($S_B = 10$); (– · ·–) density-maximum fluid ($S_N = 10$).

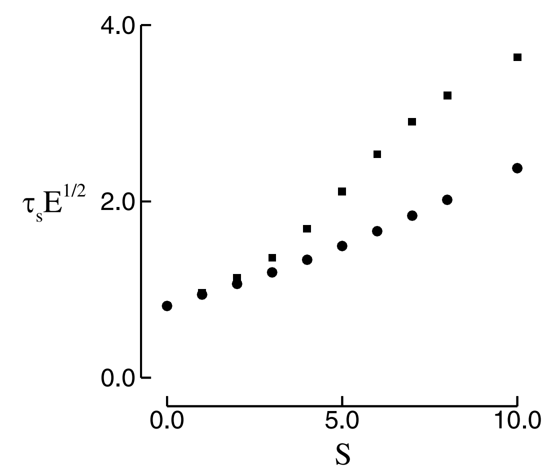

Fig. 7. Spin-up time τ_s. S stands for the stratification number. (■) Boussinesq fluid; (●) density-maximum fluid.

time $E^{-1/2}$, is shown in Fig. 7. Here, τ_s is defined to be the time at which the global spin-up is achieved to be e^{-1} of the final-state value. Expectedly, the spin-up time τ_s

increases with the imposed stratification, and spin-up proceeds faster for a density-maximum fluid than for a Boussinesq fluid. These reconfirm the trends discussed earlier.

4. Conclusion

The present numerical results demonstrate the spin-up process of a stratified, density-maximum fluid (water), in which the temperature at the bottom disk is the density-maximum temperature, $T_B = T_m$.

The meridional flow is asymmetric about the mid-height plane, $z = 0.5$, and it is stronger in the lower half plane ($z < 0.5$) than in the upper half plane ($z > 0.5$). Spin-up proceeds more rapidly in the lower half plane than in the upper plane. This stems from the non-uniform vertical distribution of density stratification based on the quadratic ρ–T relation. The overall spin-up time for a density-maximum fluid is shorter than for a conventional Boussinesq fluid.

The present numerical results provide baseline flow data which may be used to examine the stability issues in the Ekman layers and other regions.

Acknowledgements

This work was supported by grants from KOSEF, MOST, NRL-project, South Korea.

References

Barcilon, A., Lau, J., Piacsek, S., Warn-Varnas, A., 1975. Numerical experiments on stratified spin-up. Geophys. Fluid Dyn. 7, 29–42.

Braga, S.L., Viskanta, R., 1992. Transient natural convection of water near its density extremum in a rectangular cavity. Int. J. Heat Mass Transfer 35, 861–875.

Buzyna, G., Veronis, G., 1971. Spin-up of a stratified fluid. J. Fluid Mech. 50, 579–608.

Dolzhanskii, F.V., Krymov, V.A., Manin, D.Yu., 1992. Self-similar spin-up and spin-down in a cylinder of small ratio of height to diameter. J. Fluid Mech. 234, 473–486.

Greenspan, H.P., Howard, L.N., 1963. On a time-dependent motion of a rotating fluid. J. Fluid Mech. 17, 385–404.

Greenspan, H.P., 1980. A note on the spin-up from rest of a stratified fluid. Geophys. Astrophys. Fluid Dyn. 15, 1–5.

Hayase, T., Humphrey, J.A.C., Greif, R., 1992. A consistently formulated QUICK scheme for fast and stable convergence using finite-volume iterative calculation procedures. J. Comput. Phys. 98, 108–118.

Hyun, J.M., Fowlis, W.W., Warn-Varnas, A., 1982. Numerical solutions for the spin-up of a stratified fluid. J. Fluid Mech. 117, 71–90.

Kwak, H.S., Kuwahara, K., Hyun, J.M., 1998. Convective cool-down of a contained fluid through its maximum density temperature. Int. J. Heat Mass Transfer 41, 323–333.

Moore, D.R., Weiss, N.O., 1973. Nonlinear penetrative convection. J. Fluid Mech. 61, 553–581.

Nishimura, T., Wake, A., Fukumori, E., 1995. Natural convection of water near the density extremum for a wide range of Rayleigh numbers. Numer. Heat Transfer 27, 443–449.

Park, J.S., Hyun, J.M., 2001. Transient response of a compressible fluid in a rapidly rotationg circular pipe. J. Fluid Mech. 427, 275–297.

Patankar, S.V., 1980. Numerical Heat Transfer and Fluid Flow. McGraw-Hill, New York.

Robillard, L., Vasseur, P., 1982. Convective response of a mass of water near 4 °C to a constant cooling rate applied on its boundaries. J. Fluid Mech. 118, 123–141.

Sakurai, T., 1969. Spin-down problem of rotating stratified fluid in thermally insulated circular cylinders. J. Fluid Mech. 37, 689–699.

Saunders, K.D., Beardsley, R.C., 1975. An experimental study of the spin-up of a thermally stratified rotating fluid. Geophys. Fluid Dyn. 7, 1–27.

Walin, G., 1969. Some aspects of time-dependent motion of a stratified rotating fluid. J. Fluid Mech. 36, 289–307.

Warn-Varnas, A., Fowlis, W.W., Piacsek, S., 1978. Numerical solutions and laser-Doppler measurements of spin-up. J. Fluid Mech. 85, 609–639.